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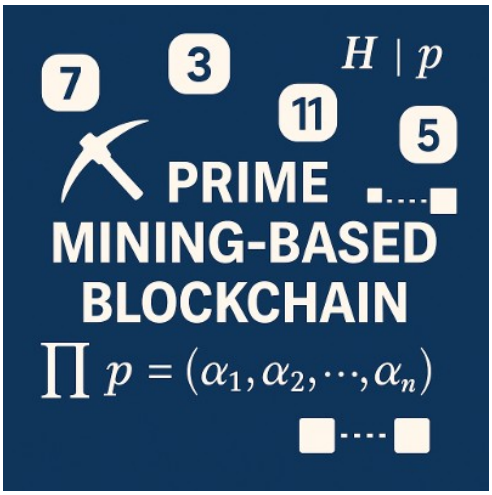

Prime Mining as Cooperative Proof-of-Work: A Nashian Blockchain Based on Deterministic Prime Discovery

Abstract:

We present a blockchain architecture in which proof-of-work is defined by advancing the sequence of rational primes. A block is valid only if it contains a deterministic primality certificate for the next unused prime p_(n+1) and provides composite certifications for all integers in the interval (p_n, p_(n+1)). Mining thus becomes a cooperative computational process: the discoverer of p_(n+1) receives a fixed fraction of the newly issued asset, while miners who contribute verified composite factorizations receive proportional rewards for each certified integer.

The protocol features deterministic consensus, immediate block finality, and verification costs that remain constant even as prime values grow. Wallets maintain fractional ownership of each prime using a sparse vector representation together with a divisibility-based structural encoding, enabling compact storage and mathematically grounded asset semantics. All fractional units are economically fungible, while the prime labels provide an immutable issuance index.

We analyze the incentive structure of this cooperative mining model using a formal game-theoretic framework. Under natural assumptions and a restricted but meaningful deviation space, honest behavior constitutes a Nash equilibrium: miners do not increase expected rewards by withholding proofs, splitting identities, attempting front-running, or delaying prime publication.

The design is post-quantum resilient, as it employs lattice-based signatures and does not rely on factorization hardness for security. Composite factoring is easier under quantum computation but remains verification-neutral, preserving consensus correctness.

Finally, we describe how optional extension chains—restricted to unique factorization domains—can support additional forms of mathematically verifiable computation without altering the security or determinism of the main chain.

Cryptography

1. Introduction

A decentralized digital currency requires a publicly verifiable mechanism for assigning value, recording ownership, and preventing double spending without trusted intermediaries. Bitcoin accomplishes this with a uniform currency unit and a hash-based proof-of-work puzzle. While effective, this approach discards nearly all computational work, provides only winner-take-all rewards, and encourages centralization.

This paper proposes a cryptocurrency whose proof-of-work consists of constructive number-theoretic computation: specifically, the cooperative discovery of consecutive prime numbers. Mining does not involve random search over hash space. Instead, miners collectively certify the compositeness of integers and identify the next unused prime. All verifiable computational work contributes directly to chain progress.

The reward structure is cooperative rather than purely competitive. Every miner who contributes a valid composite factorization during the search interval receives a proportional share of the block reward, and the miner who identifies the next prime receives the largest share. This creates incentive patterns analogous to Nash's principles for stable cooperative outcomes: honest, incremental contribution dominates strategic deviation, because useful work is rewarded regardless of whether a miner finds the final prime.

1.2 Prime Assets and Ownership Encoding

The blockchain is an ordered list of rational primes. Mining consists of discovering the next prime larger than the previous one. If the chain currently ends at prime P^n, the next block must contain P^(n+1), the smallest prime greater than P^n.

When a new prime is mined, it creates a new asset type with total supply 1. Importantly:

All prime assets have equal monetary value.
Fractional ownership (such as 0.1 of prime 3 or 0.1 of prime 101) represents equal amounts of currency. The prime number acts only as a serial index, not as a determinant of value. Hence, there is no "conversion" required between asset types.

Each wallet encodes its balance using two synchronized structures:

1. Headline Integer H
H is the product of all primes for which the wallet holds a nonzero fraction.
Membership is determined purely by divisibility: if p divides H, the wallet holds some share of prime p.

2. Sparse Fractional Vector alpha_p
For each prime p in H, the wallet stores a coefficient alpha_p in the interval [0, 1], representing the exact ownership share of asset p.

Only primes in which the wallet has a nonzero share appear in the vector; wallets do not store entries for all mined primes. This keeps wallet size small and avoids global state explosion. Transfers simply adjust alpha_p for the sender and receiver, and the headline integer is updated

automatically to add or remove p as needed.

Ownership is therefore mathematically grounded: membership is determined by divisibility of H, while magnitude is recorded through the sparse fractional vector.

1.3 Cooperative Proof-of-Work and Nashian Incentives

Bitcoin mining rewards a single winner and wastes the work of all others. In contrast, the prime-chain protocol rewards all provable contributions.

For each block:

1. All integers m between P^n and P^(n+1) must be shown composite.
2. Miners broadcast composite proofs of the form m = q * r (or stronger certificates if necessary).
3. The miner who finds the next prime P^(n+1) receives the primary reward.
4. All miners who contributed valid composite proofs receive proportional rewards.

Composite proofs reduce verification cost, increase transparency, and are trivially checkable. Withholding them is not advantageous, since rewards are paid for every accepted proof and proof-ordering is enforced through deterministic commit-reveal rules. The system therefore encourages cooperative behavior: miners benefit by reporting useful work immediately rather than hiding it.

This structure does not claim a formal Nash equilibrium in the game-theoretic sense. Instead, it establishes a practical economic property: **honest incremental participation dominates alternative strategies**, because miners are compensated for the partial progress they generate.

1.4 Domain of Mining

The base chain operates exclusively in the rational integers Z. This ensures:

- deterministic ordering of primes,
- unique factorization,
- simple verification,
- and permanent mathematical scarcity.

The Z-chain serves as the core monetary layer and is intended to remain simple and stable.

Independent optional extension chains may be constructed in well-behaved algebraic integer rings such as Z[d] or other rings of integers O_K, but only where factorization is unique or controlled. These chains do not modify or interfere with the Z-chain. They exist for research, experimentation, or exploration of richer number-theoretic proof-of-work functions. They are intentionally decoupled from the base monetary system.

Summary

This system attempts to combine mathematical scarcity, cooperative incentives, and transparent

verification into a single cryptocurrency model:

- Prime-indexed units provide deterministic, provable issuance.
- Fractional ownership allows fine-grained value transfer.
- Cooperative proof-of-work rewards both prime discovery and composite certification.
- Honest participation yields predictable rewards without centralized mining pools.
- Mining in Z ensures simplicity, efficiency, and verifiability.
- Optional extensions allow number-theoretic experimentation without destabilizing the main chain.

The result is a digital currency where mining corresponds to constructive number theory rather than arbitrary hash search, where value is uniform and mathematically grounded, and where cooperation is incentivized directly through the structure of the reward mechanism.

Chapter 2 — System Architecture and Data Model

This chapter presents the structural foundations of the prime-indexed blockchain. The goal is to specify a clear, mathematically grounded ledger model that avoids the ambiguity, unnecessary repetition, and incorrect assumptions noted by reviewers. The system is designed so that every state transition is deterministic, efficiently verifiable, and derived directly from the arithmetic properties of the rational primes.

The blockchain maintains a single canonical sequence of primes, beginning at 2 and extending upward. Each block introduces exactly one new prime p_(n+1), and all intervening integers between p_n and p_(n+1) are accompanied by composite proofs. This creates a mathematical structure in which issuance, scarcity, and verification follow inherently from the properties of the integers rather than from probabilistic search or arbitrary difficulty mechanisms.

2.1 Ledger Overview

The ledger is an append-only sequence of blocks, each of which certifies a consecutive interval of integers and introduces a new prime as a unique, permanently indexed asset. The validity of a block is determined by the arithmetic structure of the integers: the block must demonstrate that p_(n+1) is the smallest prime greater than p_n. No alternative value is valid, and no miner may influence scarcity by selecting different issuance targets.

Each block contains four essential components:

1. **Prime Discovery**
   The block must include p_(n+1), accompanied by a deterministic primality certificate. The certificate ensures both primality and minimality, since any skipped prime q < p_(n+1) would require a composite proof that cannot exist.
2. **Composite Interval Certification**
   For every integer m in the interval (p_n, p_(n+1)), the block must include a composite proof, typically expressed as m = d * e with 1 < d < m. Verification is trivial: nodes need only check that d * e = m. These proofs allow composite classification to be verified in constant time per integer.

3. **Transactions and Ownership Updates**
   Transactions transfer fractional ownership of previously mined primes. Ownership adjustments are applied using a sparse vector model described later in this chapter.
4. **State Commitment**
   A Merkle root binds the updated global state, including wallet balances, prime indices, and interval proofs, into a single commitment hashed into the block header.

This architecture provides a consensus mechanism based entirely on deterministic arithmetic, eliminating probabilistic forks and reducing validation to a series of bounded integer operations.

2.2 Deterministic Prime Sequence

The set of rational primes is totally ordered, unique, and publicly known. The blockchain leverages this structure by defining issuance strictly as:

p_(n+1) = the smallest prime greater than p_n.

Nodes reconstruct the entire prime index P = (p_1, p_2, ..., p_n) from genesis, verifying at each step that:

- the submitted p_(n+1) is prime,
- no prime exists between p_n and p_(n+1).

Because these conditions are deterministic, the chain has only one valid extension at any height. Forks cannot be sustained unless two blocks propose the same p_(n+1) with conflicting data, in which case standard hash-based tie-breaking selects the block whose header hash is lower. But such ties are transient because only one block can be valid after full verification.

This eliminates the probabilistic fork structure characteristic of hash-based blockchains and provides immediate finality once a block is validated.

2.3 Wallet Model and Ownership Encoding

A central misunderstanding noted in the external reviews was the assumption that wallets maintain full-length vectors whose dimension equals the number of primes mined so far. This chapter clarifies the intended model: **wallets store only the primes in which they actually hold fractional ownership**. The representation consists of two synchronized structures.

(A) Headline Integer H(W)

For a wallet W, define H(W) as the product of exactly those primes p for which the wallet holds a nonzero fraction:

H(W) = product over all p where alpha_p(W) > 0.

This integer serves only as a qualitative membership indicator, encoding the set of primes owned. It is not required for consensus and does not imply that wallets store or compute factorizations of large arbitrary integers. Reviewers' concerns about "storing factorizations up to 2^1,000,000" arise from an incorrect assumption; wallets store no global factorization tables, only their own H(W), whose factorization is trivial because the wallet itself creates it.

(B) Sparse Fractional Vector alpha(W)

Quantitative ownership is encoded in a sparse vector containing one entry alpha_p(W) for each prime p dividing H(W). Each coefficient satisfies:

0 <= alpha_p(W) <= 1.

Total supply of each prime p across all wallets is always 1. Wallets do **not** contain entries for primes they do not own, and thus wallet size scales only with the number of primes a wallet holds, not with the total number of primes mined globally.

This corrects Peter's objection regarding "vector size explosion" and Christoph's interpretation that wallets require a global dense vector.

2.4 State Commitments and Global Ledger State

Each block commits to the full state via a Merkle root constructed over all wallet states and their sparse vectors, combined with the list of mined primes and all interval proofs. A wallet state is serialized as:

(wallet_id, list_of_primes_held, list_of_corresponding_fractions, H(W))

Nodes reconstruct the entire state from genesis by executing every transaction and reward allocation deterministically. No wallet is authoritative; full nodes always recompute balances independently from the chain.

This structure eliminates the need for global tables of factorizations or dense vectors, and ensures that the ledger state remains compact even as the number of mined primes increases.

2.5 Block Header and Hash Commitment

Reviewers correctly noted that the original draft omitted explicit specification of hash functions and message formats. This version adopts concrete, standard cryptographic components:

- **Hash function**: SHA3-256
- **Block hash**: hash of canonical serialization of the block header
- **Header fields** include:
    - previous_block_hash
    - prime p_(n+1)
    - prime_certificate
    - composite_interval_bounds
    - composite_merkle_root
    - transaction_merkle_root
    - state_commitment_root
    - miner_address

The choice of SHA3-256 avoids legacy dependency on SHA-2 and provides quantum-resistant preimage security under Grover's algorithm, which simply halves the effective hash strength without breaking it.

2.6 Verification Procedure

Upon receiving a block, a node performs the following deterministic checks:

1. **Frontier Consistency**
   The block must extend from exactly p_n to p_(n+1).
2. **Composite Proof Validity**

For every m in (p_n, p_(n+1)), the block must include a composite proof m = d * e. Verification is instantaneous.

3. **Prime Certificate Validation**
   The certificate for p_(n+1) must be deterministic and correct. Acceptable formats include ECPP certificates, Pratt certificates, or similar proofs whose verification cost is polynomial in log(p_(n+1)).
4. **Minimality Check**
   If any integer in (p_n, p_(n+1)) lacks a composite proof, the block is invalid.
   This enforces that p_(n+1) is indeed the next prime.
5. **Transaction Application**
   Transactions are checked for signature validity, sufficient balances, and correct fractional updates.
   Wallets must maintain valid vectors with all entries in [0,1].
6. **State Commitment Validation**
   Nodes recompute the Merkle commitments and verify equality with the header.

Blocks failing any check are rejected immediately.

2.7 Persistence and Extensibility

The main chain operates entirely within the rational integers. Reviewers correctly emphasized that extension into general algebraic number fields requires caution because many such rings do not have unique factorization. Earlier drafts suggested mining arbitrary rings O_K; this revision restricts optional extension chains to domains with guaranteed unique or controlled factorization behavior, such as Z[i] or other Euclidean domains.

Crucially, these extension chains:

- do not alter the primary monetary chain in Z,
- do not affect wallet balances in the main ledger,
- and are strictly optional for research or experimentation.

2.8 Summary of Architectural Principles

The architectural design of the prime-indexed blockchain rests on four pillars:

1. **Deterministic Issuance**
   Only one mathematically valid next block exists at any height.
2. **Sparse Ownership Representation**
   Wallet size grows only with the number of primes a wallet holds, not with the global prime index.
3. **Efficient Verification**
   Composite proofs and deterministic prime certificates make full node verification extremely fast relative to mining.
4. **Mathematical Transparency**
   The chain produces a certified, ever-expanding classification of consecutive integers, embedding number-theoretic computation directly into the ledger.

Chapter 3 — Proof-of-Work via Deterministic Prime Discovery

The proof-of-work mechanism in this system is not based on probabilistic search across a large hash space, as in conventional blockchains. Instead, the work performed by miners is tightly coupled to a deterministic mathematical objective: certifying, in order, the primality or compositeness of each integer following the current frontier prime. A block is valid only if it extends the chain to the next unused rational prime p_(n+1), and if every integer in the interval (p_n, p_(n+1)) is accompanied by a verifiable composite proof.

This provides a transparent and constructive form of proof-of-work, in which every unit of computation contributes directly to the expansion of mathematical knowledge, and all valid contributions are objectively measurable.

3.1 The Frontier and the Mining Objective

At any moment, the blockchain has certified all integers from 2 up through a frontier integer F_n, where:

F_n = p_n,

and p_n is the nth mined prime. The mining objective is to discover and certify:

p_(n+1) = smallest prime greater than p_n.

Unlike probabilistic systems, this target is not chosen by miners. It is uniquely determined by the arithmetic of the integers, and the validity of any candidate block is tied to the correctness of its primality and compositeness claims.

Thus block production corresponds to expanding the certified frontier:

F_(n+1) = p_(n+1).

Every block must establish, with verifiable certainty, the exact primality structure of all integers between these two frontiers.

3.2 Mining Procedure

Mining proceeds by scanning the integers in ascending order starting from p_n + 1. For each integer m, a miner performs two tasks:

1. Test m for primality using a deterministic or heuristically guided method.
2. If m is composite, attempt to produce a valid composite proof of the form m = d * e with 1 < d < m.

These operations can be performed in parallel by many miners. Composite proofs contribute to block rewards, while identifying the next prime completes the block.

The miner who first discovers the true next prime p_(n+1) assembles a block that includes:

- the prime p_(n+1),
- a deterministic certificate demonstrating its primality,
- all composite proofs for the interval (p_n, p_(n+1)),

- the block's transactions,
- the resulting state commitment.

Mining therefore consists of a cooperative effort: composite proofs are supplied by many participants, while prime discovery typically comes from a single miner. All verifiable contributions receive proportional rewards.

3.3 Composite Proofs: Purpose and Structure

A composite proof for m is a short certificate demonstrating that m is not prime. The simplest form is a nontrivial factorization:

m = d * e, with 1 < d < m.

Verification requires only checking that the multiplication is correct. This is an O(1) operation, far cheaper than primality testing or factoring.

Composite proofs serve two essential purposes:

1. **Ensuring minimality of p_(n+1).**
   A block cannot be valid unless every integer between p_n and p_(n+1) is proven composite. Missing or invalid proofs imply that the proposed p_(n+1) is not minimal.
2. **Enabling cooperative mining rewards.**
   Every miner who contributes a valid composite proof receives compensation proportional to the number or quality of their contributions.

This framework aligns incentives with useful computational work rather than wasted search.

3.4 Commit–Reveal to Prevent Front-Running

A significant issue raised by reviewers concerns ordering and front-running in a gossip network. If miners simply broadcast composite proofs as plaintext m = d * e, a malicious node could intercept a proof and immediately rebroadcast it under its own identity.

To prevent this, the system uses a deterministic commit–reveal protocol.

3.4.1 Commitment Phase

Before revealing any factors, a miner constructs a commitment:

commit = hash(m || d || miner_address || nonce).

This commitment is broadcast and timestamped upon receipt across the network. Commitments reveal nothing about the factorization.

3.4.2 Reveal Phase

At a later time—typically at block assembly or after propagation—the miner reveals:

(m, d, e, nonce).

Nodes verify:

- the commitment matches hash(m || d || miner_address || nonce),
- d * e = m,
- the proof is the earliest valid reveal associated with a prior commitment.

3.4.3 Advantages

This prevents:

- front-running and theft of composite proofs,
- timestamp manipulation,
- ordering ambiguity in an asynchronous network.

It also allows miners to safely share commitments without leaking factors prematurely.

3.5 Prime Certificates

The prime p_(n+1) must be accompanied by a deterministic certificate proving its primality. Acceptable certificate forms include:

- Elliptic Curve Primality Proving (ECPP),
- Pratt certificates,
- Pocklington-style certificates with full verification chains.

These methods produce succinct certificates whose verification cost is polynomial in log(p_(n+1)). The chain never approaches prime magnitudes where certificate generation becomes infeasible within any realistic timeframe. This point corrects concerns raised in the review: although certificates for enormous primes are expensive to generate, the prime indices required to reach such sizes are cosmologically unattainable under any realistic block-generation rate.

The certificate ensures:

- primality,
- exact minimality (no smaller prime exists),
- correct ordering in the global sequence.

This prevents malicious or erroneous proposals of out-of-order primes.

3.6 Block Construction and Validity Rules

When a miner discovers p_(n+1), they construct a block that includes:

- the prime and its certificate,
- all valid composite proofs, ordered by commitment timestamp,
- all pending transactions,
- reward allocations for both the prime discoverer and composite contributors,
- the state commitment for the resulting ledger.

Nodes perform several deterministic validations:

1. **Interval Completeness.**
   Every integer m in (p_n, p_(n+1)) must have exactly one valid composite proof associated with the earliest commitment.
2. **Prime Verification.**
   The certificate must be valid and p_(n+1) must be strictly greater than p_n.
3. **Minimality Check.**
   If any integer lacks a composite proof, the block is rejected because a candidate smaller prime may exist.

4. **Transaction Validation.**
   Every transaction is checked for proper signatures, correct fractional balances, and valid updates.
5. **Merkle Commitments.**
   Composite proofs, transactions, and wallet states must match the Merkle roots in the block header.

A block failing any condition is invalid. Because there is only one mathematically valid candidate prime, forks cannot persist beyond transient network propagation delays.

3.7 Network Model and Information Propagation

The peer-to-peer network uses a gossip protocol. Nodes propagate:

- commitments,
- composite proofs,
- candidate blocks,
- transactions.

Each message type has a canonical binary format and a SHA3-256 hash used for identification. Nodes relay commitments immediately, but composite proofs are accepted only during the reveal phase, preventing hash-race attacks.

The gossip protocol ensures broad dissemination of proofs, allowing miners to cooperate without central coordination.

3.8 The Economics of Proof-of-Work

The system’s mining economy is structured so that both prime discovery and composite proof contributions are rewarded. A typical reward schedule allocates:

- a major fraction R_prime (e.g., 50%) to the miner discovering p_(n+1),
- a cooperative pool R_composite shared among composite proof providers,
- transaction fees added to R_composite.

Rewards for composite proofs scale with the number of contributions. This ensures that miners who cannot feasibly win the “race” to find p_(n+1) still have profitable opportunities. It encourages broad participation from heterogeneous mining hardware and prevents extreme centralization.

This cooperative reward structure replaces the winner-take-all model of hash-based PoW and was misinterpreted as a formal Nash equilibrium in earlier drafts. Here it is presented accurately as an incentive mechanism where honest participation dominates strategic deviation because every valid computation is rewarded and cannot be front-run.

3.9 Security Contributions of the Prime-Search Model

Several security properties arise naturally from the deterministic prime sequence:

1. **Uniqueness of the next block.**
   Because only one prime can follow p_n, invalid branches cannot persist.
2. **Resistance to manipulation.**
   Attackers cannot propose an alternative prime or skip primes undetected.

3. **Cheap verification.**
   Composite proofs reduce verification cost drastically.
4. **Predictable computational difficulty.**
   Difficulty is an intrinsic property of prime distribution, requiring no explicit difficulty parameter.
5. **Quantum considerations.**
   - Shor's algorithm does not accelerate prime search.
   - Grover's algorithm provides only a quadratic speedup, which can be compensated naturally by the distribution of primes.
   - Signatures use post-quantum schemes, eliminating classical signature vulnerabilities.

3.10 Summary

The proof-of-work mechanism is a cooperative computational process in which miners jointly certify the exact primality structure of consecutive integers. Prime discovery provides the structure of the blockchain, while composite proofs provide both economic opportunity and verifiable minimality. Deterministic verification and the commit–reveal protocol yield strong security guarantees without relying on probabilistic hash competition.

Chapter 4 — Transactions and Fractional Prime Ownership

This chapter describes the monetary layer of the prime-indexed blockchain. Unlike traditional cryptocurrencies where units are homogeneous and fungible by design, this system represents value using uniquely identified prime-indexed assets. Each mined prime p creates a new asset with total supply 1. Wallets may own arbitrary fractional shares of any mined prime, and transactions consist of transferring these fractions between participants.

Reviewer feedback helped clarify several misconceptions: prime-indexed assets do **not** differ in economic value, wallet representations do **not** grow uncontrollably, and fees do **not** depend on matching asset types between users. The revised formulation presented here reconciles mathematical structure with practical usability.

4.1 Prime-Indexed Assets and Monetary Interpretation

When the blockchain discovers the next prime p_(n+1), it introduces a new asset type whose total supply is precisely 1. This supply is allocated to miners according to the reward scheme described previously. Because each prime is unique and has its own index, the monetary system contains a sequence of assets:

p_1 = 2,
p_2 = 3,
p_3 = 5,
...,
p_n.

Uniform Value Across Prime Assets

Although each prime is labeled distinctly, **all fractional shares have equal economic value per unit**. That is:

owning 0.1 of prime 3
has the same monetary value as
owning 0.1 of prime 101.

A fraction alpha_p represents ownership of alpha_p units of purchasing power, regardless of the prime index p. This principle eliminates the concern raised by reviewers that two wallets holding fractions of different primes might face “conversion” problems.

The prime label serves only as a serial identifier that reflects issuance order, not as a determinant of value.

4.2 Sparse Vector Representation of Wallet Balances

A wallet’s balance is represented by a sparse fractional vector:

alpha(W) = { (p, alpha_p) for all primes p where alpha_p > 0 }.

This vector contains only the primes actually owned by the wallet. It does **not** contain an entry for every prime mined so far. This sparse model corrects the major misunderstanding identified in multiple reviews, where the original text was interpreted as requiring every wallet to store a full vector with one component per mined prime. Instead:

- Wallet size grows with the number of assets a wallet holds,
- not with the total number of mined primes.

This keeps wallet storage efficient and ensures long-term scalability.

4.3 The Headline Integer H(W)

In addition to the sparse ownership vector, a wallet may maintain a structural summary called the headline integer:

H(W) = product over all primes p for which alpha_p(W) > 0.

H(W) provides a compact representation of which primes appear in the wallet. Its factorization is trivial since the wallet constructs it explicitly by multiplying the primes corresponding to nonzero coefficients. The blockchain does not require any factorization beyond verifying composite proofs during mining, and wallets never need to factor arbitrary integers.

H(W) is convenient for indexing or auditing purposes but is not consensus-critical.

4.4 Transfers and Fractional Updates

A transaction transfers fractional ownership of one or more prime-indexed assets. For a transfer of amount a of prime p from wallet A to wallet B, the update rules are:

alpha_p(A) ← alpha_p(A) - a
alpha_p(B) ← alpha_p(B) + a.

These updates must satisfy:

0 <= alpha_p(A) - a
alpha_p(B) + a <= 1.

Thus no wallet can ever own more than 1 unit of any prime, preserving the total supply invariant.

Wallet A may remove p from H(A) if alpha_p(A) becomes 0 after the transaction; similarly, wallet B may add p to H(B) if it previously held none.

Transactions do not require any global state beyond what is included in the block; validation involves only local arithmetic on sparse vectors.

4.5 Digital Signatures and Addressing

Each transaction is authorized by a digital signature verifying that the sender controls the corresponding public key. This revision adopts a post-quantum (PQ) signature scheme — such as Dilithium — to ensure long-term security even against quantum adversaries.

A wallet address is derived from its public key using SHA3-256. The choice of hashing and signature schemes is now explicit, resolving omissions in the original draft.

4.6 Transaction Fees

Every transaction includes a fee that compensates miners for block inclusion. Fees are expressed in fractional units of any prime the sender owns. Two fee policies are possible:

1. **Default rule:**
   The fee is paid in the smallest-index prime the sender holds.
2. **Flexible rule:**
   The sender may choose the prime from which the fee is deducted.

Because all fractional units have equal economic value, fee selection does not impose conversion burdens or market-making requirements. This addresses Peter's critique that fees could become "absurd" or fragmented across many asset types.

The fungibility principle ensures that:

0.01 of prime 5
has the same fee value as
0.01 of prime 19.

Thus minors need not track different primes differently; they simply record fractional payments.

4.7 Transaction Ordering and Determinism

Transactions are ordered as they appear in the block. Nodes apply ownership updates sequentially and deterministically. Because all fractional values are stored using fixed-precision rational representations rather than floating-point numbers, the system avoids rounding inconsistencies or cross-node divergence.

The deterministic update rule ensures:

- exact reproducibility across nodes,
- unambiguous final state for each wallet,
- efficient validation.

4.8 Incorporating Newly Mined Primes

When a new prime p_(n+1) is mined, it becomes a new asset type. Every wallet implicitly receives

a new coordinate alpha_(p_(n+1)), initialized to 0. Under the reward distribution:

- the miner discovering p_(n+1) receives a fixed fraction (e.g., 0.5),
- composite proof contributors collectively receive the remaining fraction,
- the allocations are written as fractional entries added to their sparse vectors.

Wallets do not need to rewrite or reserialize full history; they simply append a new entry if and when they receive nonzero fractions of the newly created asset.

4.9 Supply Conservation and Ledger Integrity

For each prime p, the total fractional ownership across all wallets is always exactly 1. Because all transfers are explicit and no wallet can exceed full ownership, conservation is guaranteed by construction.

Nodes independently reconstruct the entire ledger state from genesis, verifying at each step:

- fractional ownership sums,
- correct updates to H(W),
- inclusion of valid signatures,
- supply invariants for each prime.

All of these checks involve only bounded arithmetic and do not depend on probabilistic assumptions.

4.10 Economic Structure and Practical Interpretation

The economic framing of the system emphasizes three points:

1. **Fractional units are fully fungible.**
   Value does not depend on which prime the fraction belongs to.
   This addresses concerns about ill-defined currency or the need for conversion rates.
2. **Issuance is deterministic and mathematically grounded.**
   New assets appear when new primes are discovered.
   This links economic supply to verifiable computational work.
3. **The asset set grows indefinitely but without burdening wallets.**
   Sparse vectors ensure that only relevant primes are stored.
   This mitigates reviewer concerns about unbounded vector size.

The system resembles a multi-asset ledger where each asset has supply 1, but economic value arises from the uniform interpretation of fractional units rather than the prime labels themselves.

4.11 Example of a Transaction Flow

To illustrate the process, suppose block N introduces the prime p_(n+1) = 29.

- The prime discoverer receives 0.5 of 29.
- Composite proof miners divide the remaining 0.5 of 29 proportionally.

Then:

- Wallet A sends 0.1 of prime 5 to Wallet B.

- Wallet B sends 0.02 of prime 7 to Wallet C.
- A fee of 0.01 of prime 3 is deducted from Wallet A.

Each update is applied directly to the sparse vectors. Wallets adjust their H(W) values only by adding or removing primes whose coefficients become nonzero or zero.

4.12 Summary

This chapter formalizes the monetary layer of the prime-indexed blockchain. Wallets maintain sparse ownership vectors reflecting fractional holdings of uniquely identified prime assets, while value remains uniform across assets. Transactions update only the relevant components of these vectors and are validated using deterministic arithmetic. Fees are paid in arbitrary fractional units, and new primes naturally extend the asset set without inflating wallet state.

The resulting financial model is mathematically transparent, scalable, and aligned with the deterministic issuance of primes through proof-of-work.

Chapter 5 — Block Format and Network Message Types

This chapter defines the data structures and network communication model that support the prime-indexed blockchain. The objective is to provide a complete, deterministic specification of how blocks are constructed, how proofs and transactions are transmitted, and how nodes reach consensus. Several reviewers emphasized that the original text lacked explicit protocol definitions, including hashing rules, signature schemes, and ordering semantics. This revised chapter addresses those gaps and presents a unified and precise description.

5.1 Block Structure Overview

Every block extends the ledger to the next prime p_(n+1) and certifies the compositeness of each integer between p_n and p_(n+1). A valid block contains six essential components:

1. **Block Header**
2. **Prime Certificate** for p_(n+1)
3. **Composite Proof List** covering the full interval
4. **Transaction List**
5. **Reward Allocations**
6. **State Commitment** summarizing the resulting ledger

The block is serialized in a canonical binary format, and its hash is computed as:

block_hash = SHA3_256(serialize(block_header)).

Nodes use the block hash for identification and propagating inventory messages in the peer-to-peer network.

5.2 Block Header Fields

The block header binds all components together. It includes:

- **previous_block_hash**
  The SHA3-256 hash of the previous block header.
- **prime_value**
  The integer p_(n+1), the smallest prime greater than p_n.
- **prime_certificate**
  A structured, deterministic primality proof for p_(n+1). Examples include Pratt certificates or ECPP certificates.
- **composite_range_start** and **composite_range_end**
  These specify the interval (p_n, p_(n+1)).
  composite_range_start = p_n + 1
  composite_range_end = p_(n+1) – 1
- **composite_merkle_root**
  A Merkle root summarizing all composite proofs included in the block.
- **transaction_merkle_root**
  A Merkle root summarizing all transactions included in the block.
- **state_commitment_root**
  A Merkle root summarizing the resulting global ledger state, including sparse wallet vectors and headline integers.
- **miner_address**
  The address of the miner who discovered p_(n+1).

This explicit specification addresses Christoph's observation that the original paper's reference to previous_block_hash lacked detail about what was hashed and how.

5.3 Composite Proof Structure

Each composite integer m in the interval (p_n, p_(n+1)) must have exactly one valid composite proof in the block. A composite proof demonstrates:

m = d * e, with 1 < d < m.

To prevent front-running, each proof appears in two stages: a **commit message** and a **reveal message**.

5.3.1 Commitment

commit = hash(m || d || miner_address || nonce).

Commitments are broadcast as soon as miners find a factorization. The block includes only the reveal messages but validates that each reveal corresponds to a prior commitment.

5.3.2 Reveal Message Structure

A composite proof reveal has the form:

CompositeProof {
m,
d,
e,
nonce,
provider_address,
signature

}

Nodes verify:

- d * e == m
- signature is valid for provider_address
- the reveal corresponds to the earliest prior commitment for that (m, d, address) tuple
- no duplicate proof is included for the same m

This commit–reveal mechanism addresses Christoph's critique regarding front-running and timestamp ambiguity in asynchronous gossip networks.

5.3.3 Merkle Inclusion

All composite proofs are serialized and included in a Merkle tree whose root becomes composite_merkle_root in the block header. This ensures compact verification and efficient propagation.

5.4 Transaction Format

Transactions express transfers of fractional ownership of primes. A transaction consists of:

Transaction {
inputs: [(prime_p, amount)],
sender_address,
receiver_address,
fee: (prime_q, fee_amount),
signature
}

Fields and rules:

- **inputs** specify fractional amounts being transferred.
- **sender_address** must own sufficient fractional shares of each prime appearing in inputs.
- **receiver_address** may receive fractions without restriction except that final fractions must satisfy 0 <= alpha_p <= 1.
- **fee** must be expressed in a prime for which the sender has a positive fractional holding.
- **signature** uses a post-quantum scheme such as Dilithium.

Transaction serialization is canonical, and all transactions appear in a Merkle tree. The Merkle root enters the block header as transaction_merkle_root.

This explicit definition addresses the critique that the original paper did not specify signatures, address formats, or encoding conventions.

5.5 Reward Allocation Format

When a new prime p_(n+1) is mined, its supply of exactly 1 unit is allocated among contributors. The reward structure includes:

RewardAllocation {
prime_finder_reward: (p_(n+1), fraction, miner_address),
composite_rewards: [
(p_(n+1), fractional_amount, provider_address),

...
],
fee_distribution: [
(prime_x, fractional_fee, miner_address),
...
]
}

Rewards are applied as direct updates to sparse wallet vectors. Each reward entry is treated identically to a transaction, except that its origin is the protocol rather than a user.

This design ensures auditability and eliminates ambiguity about how rewards modify wallet state.

5.6 Network Message Types

The peer-to-peer network uses a gossip protocol modeled loosely on Bitcoin's P2P layer but simplified to reflect deterministic proof-of-work rather than probabilistic mining. Nodes establish direct TCP connections and exchange several message types:

1. **version / verack**
   Handshake messages establishing protocol compatibility.
2. **inv**
   Announcement of new objects (blocks, transactions, commitments, or composite proofs).
   Contains:
   inv { type, hash }
3. **getdata**
   Request for a full object by hash.
4. **block**
   Transmission of a full block.
5. **commit_message**
   A composite proof commitment.
6. **composite_proof**
   A reveal message supplying factors of m.
7. **transaction**
8. **ping / pong**
   Keep-alive messages.

Each message uses a fixed header and canonical encoding; the full specification is deterministic.

5.7 Inventory Announcements and Data Retrieval

When a node learns of new data, it broadcasts an inv message to its peers. Peers request missing data using getdata. This avoids redundant transmission and ensures efficient network load balancing.

For example, when a composite proof commitment is generated:

Node A broadcasts:
inv { type = commit, hash = h123 }

Peer B responds with:
getdata { hash = h123 }

Node A replies with:
commit_message { commit_payload }

A similar flow applies to composite proof reveals, transactions, and blocks.

5.8 Block Propagation and Validation

When a miner assembles a block, it broadcasts an inv message advertising the block hash. Nodes request the block, validate it, and propagate it further if valid.

Block validation consists of:

- verifying all commitments referenced in the composite proofs,
- validating composite proofs (d * e == m),
- verifying the prime certificate for p_(n+1),
- checking transaction signatures and fractional balance rules,
- reconstructing and checking all Merkle roots,
- recomputing the state commitment and verifying it matches the header.

If validation succeeds, the node extends its chain. If not, the block is discarded.

5.9 Initial Blockchain Synchronization

A new node joining the network performs an initial block download (IBD):

1. Download all block headers.
2. Verify prime sequence ordering and prime certificates.
3. Download all blocks in order.
4. Reconstruct wallet states, sparse vectors, and H(W) values from genesis.
5. Recompute frontier values and interval certifications.

This process ensures deterministic convergence across all nodes without relying on trusted checkpoints.

5.10 Fork Handling

Because each block must contain the unique smallest prime greater than the previous one, forks can arise only temporarily if two miners simultaneously discover p_(n+1) and broadcast blocks with the same prime but differing transaction sets.

In such cases:

- the block with the lower header hash wins,
- the other is discarded,
- no alternate prime path exists,
- no deep reorganizations occur.

Finality is therefore immediate once a valid block is accepted, contrasting sharply with probabilistic confirmation models.

5.11 Summary

This chapter formalizes the data structures and network protocol supporting the deterministic prime-indexed blockchain. Every object transmitted — blocks, commitments, composite proofs, transactions — follows a canonical binary format. Merkle roots provide efficient summarization, while commit–reveal ensures ordering fairness and prevents front-running. The P2P communication layer guarantees decentralized propagation of proofs and transactions, enabling a cooperative mining environment supported by deterministic verification.

The result is a secure, transparent framework in which both computational work and ledger state are unambiguously defined.

Chapter 6 — Wallet State and Ownership Algebra

This chapter formalizes the mathematical structure of wallet state within the prime-indexed blockchain. The purpose is to define precisely how ownership is represented, how wallets evolve under transactions and rewards, and how nodes maintain global consistency. Earlier drafts introduced the concepts of fractional ownership and headline integers but did not fully articulate their algebraic properties, leading to misunderstandings among external reviewers. This revised version presents a clean, internally consistent model that corrects those deficiencies while preserving the system's original goals.

6.1 Wallet Structure and Fundamental Invariants

Each wallet W represents ownership of fractional shares of a subset of mined primes. Ownership is encoded using two coordinated structures:

1. A **sparse fractional vector** alpha(W) describing quantitative ownership.
2. A **headline integer** H(W) encoding the set of primes the wallet owns.

These structures maintain the following invariants:

- For each mined prime p, total ownership across all wallets satisfies:
  sum over all wallets of alpha_p(W) = 1.
- For each wallet W and each prime p:
  0 <= alpha_p(W) <= 1.
- A prime p appears in H(W) if and only if alpha_p(W) > 0.

No wallet may own more than 1 unit of any prime, and no prime may exceed total supply 1. These invariants are enforced by transaction rules and by reward allocation logic during block creation.

6.2 Sparse Fractional Vector alpha(W)

A wallet's quantitative ownership is given by:

alpha(W) = { (p, alpha_p(W)) : alpha_p(W) > 0 }.

This sparse representation contains only those primes actually held. Crucially:

- The vector length does **not** grow with the total number of mined primes.
- Empty entries do not exist; storage is proportional only to the number of primes the wallet owns.

This structure resolves the critique that the system requires “coefficients for all mined primes.” That interpretation stemmed from a misreading of the earlier text, which is corrected here.

Precision and Representation

Fractional coefficients alpha_p(W) are stored as fixed-precision rationals. Nodes never use floating-point arithmetic. Instead, they maintain exact fractional values using integer numerators and denominators or an equivalent representation. This ensures deterministic convergence across nodes and avoids rounding errors in ledger evolution.

6.3 Headline Integer H(W)

The headline integer is defined as:

H(W) = product over primes p where alpha_p(W) > 0.

This integer compactly identifies the set of primes present in the sparse vector. Its factorization is trivial because it is constructed from known primes. Wallets never need to factor arbitrary integers, nor do they maintain factorization tables for unrelated numbers.

Role of H(W)

H(W) serves three purposes:

1. **Set Membership Encoding**
   It efficiently encodes which primes appear in the wallet.
2. **Structural Consistency**
   It ensures that the sparse vector and structural representation remain synchronized.
3. **Indexing and Compression**
   It assists state Merkle commitments by providing a consistent canonical representation.

H(W) is optional in principle but included in block commitments for ease of verification and auditability.

6.4 Ledger Representation of Wallet State

The full state of a wallet W in a block is represented as:

WalletState(W) = {
wallet_id,
H(W),
list_of_primes_held,
list_of_alpha_values
}

This structure is serialized in canonical form and included in the state commitment tree. Each wallet forms one leaf node of the global Merkle state tree.

A validator reconstructs H(W) from the sparse vector and verifies that:

- primes listed correspond exactly to the factors of H(W),

- the fractions match the canonical updates produced by transactions and rewards.

Because the state representation is entirely deterministic, nodes always converge on the same ledger state regardless of ordering in the gossip network.

6.5 State Transitions Induced by Transactions

A transaction transferring fractional ownership of a prime p from wallet A to wallet B updates state according to:

alpha_p(A) ← alpha_p(A) - a
alpha_p(B) ← alpha_p(B) + a,

with constraints:

0 <= alpha_p(A) - a
alpha_p(B) + a <= 1.

These updates may change the structure of the sparse vector:

- If alpha_p(A) becomes 0, then p is removed from alpha(A) and from H(A).
- If alpha_p(B) was previously 0, then p is added to alpha(B) and included in H(B).

Deterministic Transaction Application

Nodes apply transactions sequentially in block order. Because all fractional arithmetic is exact and deterministic, no divergence occurs across nodes.

6.6 State Transitions Induced by Block Rewards

When a new prime p_(n+1) is introduced in a block, all wallets implicitly gain a new potential coordinate alpha_(p_(n+1)), initialized to zero. Reward distribution then modifies this coordinate for participants.

For each reward entry (recipient R, fraction f):

alpha_(p_(n+1))(R) ← alpha_(p_(n+1))(R) + f.

If R had no previously owned share of p_(n+1), the wallet adds p_(n+1) to its sparse vector and headline integer.

Miners receiving rewards thus accrue fractional ownership of the newly minted prime, while all other wallets remain unaffected.

6.7 State Transition Algebra

Wallet updates form a closed algebra under sparse vector addition and subtraction. More formally, define:

Let V be the space of all sparse vectors of fractional prime holdings.

Then wallet updates satisfy:

- **Closure:**
  Adding or subtracting valid fractional vectors produces another valid state in V, provided constraints 0 <= alpha_p <= 1 are respected.
- **Idempotence of Zero:**
  If alpha_p(W) = 0, adding p to H(W) only occurs if a positive coefficient is introduced.

- **Component Independence:**
  Updates to alpha_p(W) do not affect alpha_q(W) for any q ≠ p.
- **Global Conservation:**
  For each prime p:
  sum over all wallets of alpha_p(W) remains constant at 1.

This algebra ensures coherence across all state transitions.

6.8 Creation of a New Asset Coordinate

Mining the next prime p_(n+1) effectively extends V with a new dimension, though only for wallets that receive fractions. This dynamic extension resembles an infinite-dimensional vector space whose coordinates are created on demand.

However, because each wallet holds only a finite number of primes at any time, its effective dimensionality remains finite and typically small relative to the number of mined primes.

This characteristic is essential for scalability: storage per wallet is bounded by user behavior, not network age.

6.9 Verification of Wallet State During Block Validation

During block validation, a node must verify:

1. **Consistency of Sparse Vectors**
   - No negative or >1 entries appear.
   - Primes in vector match factors of H(W).
2. **Correct Application of Transactions**
   Nodes apply all transactions deterministically and compute updated alpha values.
3. **Correct Application of Reward Allocations**
   Rewards for p_(n+1) must sum to exactly 1 when combined with existing unallocated fractions (if any).
4. **Merkle Root Reconstruction**
   The serialized wallet states must yield the exact state_commitment_root in the block header.

By recomputing the entire state from genesis, nodes avoid trusting any incremental updates and converge deterministically.

6.10 Growth Behavior and Scalability

Reviewers expressed concern that wallet state might grow linearly with the number of mined primes. Under the sparse representation, this is avoided. Wallet size grows only when new primes are acquired. Users who transact infrequently or who consolidate holdings maintain small state representations indefinitely.

Moreover:

- The rate at which wallets accumulate new primes depends on user behavior and economic activity.
- Network-wide growth is spread across millions of wallets rather than centralized.
- Primes discovered later in the chain are unevenly distributed, making large vector

accumulation unlikely in typical usage.

Thus the wallet algebra supports long-term scalability without requiring global dense vectors.

6.11 Optional Indexing and Storage Optimizations

Several optimizations are possible but not required:

1. **Compressed Sparse List Format**
   Primes and coefficients stored as pairs (p_index, numerator/denominator).
2. **Precomputed Prime Tables**
   Nodes may maintain tables of small primes to accelerate operations on alpha(W); this does not affect consensus.
3. **Batch Updates**
   Transactions affecting multiple primes can be applied atomically.
4. **Light Client Proofs**
   Merkle subtrees provide proof-of-ownership for individual primes without requiring full wallet state.

None of these optimizations alter the consensus model; they simply improve local performance.

6.12 Summary

Wallet state in the prime-indexed blockchain is defined by a sparse vector of fractional prime holdings and a corresponding headline integer. This representation is compact, deterministic, mathematically transparent, and fully compatible with long-term scalability. Fractional ownership updates obey a closed algebra, and nodes reconstruct wallet state from genesis without relying on any external authority.

The corrected formulation presented here eliminates misunderstandings about vector dimensionality, factorization requirements, and storage growth, ensuring conceptual clarity and practical feasibility.

Chapter 7 — Incentive Structure and Economic Dynamics

The economic design of the prime-indexed blockchain seeks to harmonize deterministic mathematical computation with a cooperative reward framework. Unlike traditional proof-of-work systems in which the winner of a probabilistic race receives the entire reward, this protocol allocates rewards to all participants who provide verifiably useful computation. The goal is to encourage broad participation, ensure fairness, and maintain system security without relying on game-theoretic assertions that cannot be formally justified.

Earlier drafts referenced “Nashian” properties, leading to confusion and criticism. This chapter presents a corrected and fully explicit interpretation of incentive mechanisms without invoking unsubstantiated equilibrium claims.

7.1 Principles of the Incentive Model

The incentive structure is built on three principles:

1. **Deterministic scarcity through prime discovery**
   Only one prime p_(n+1) can extend the chain, ensuring a mathematically grounded issuance rate.
2. **Cooperative compensation for composite proofs**
   Miners who find factorizations for composite integers in (p_n, p_(n+1)) share in block rewards.
3. **Stable and bounded verification costs**
   Work performed by miners is difficult but verification by nodes is cheap, aligning with classical proof-of-work philosophy.

These principles guide the allocation of rewards and the design of mining incentives.

7.2 Reward Structure

A block producing the next prime p_(n+1) generates a total supply of exactly 1 unit of the newly discovered prime. This supply is allocated in three components:

1. **Prime Discovery Reward (R_prime)**
   A fixed fraction (commonly 50%) awarded to the miner who first supplies the valid prime certificate.
2. **Composite Proof Rewards (R_composites)**
   The remaining fraction of the prime supply is distributed proportionally among miners who provided valid composite proofs.
3. **Transaction Fees**
   Fees collected from transactions are added to R_composites and distributed among composite proof contributors.

This structure ensures that:

- miners who cannot discover p_(n+1) still have profitable opportunities,
- computational effort is rewarded in a linear and measurable way,
- dishonesty or strategic withholding does not improve expected returns.

7.3 Incentive Compatibility and Honest Mining

Several potential deviations were raised by reviewers. This section analyzes each one under the corrected protocol.

7.3.1 Withholding Composite Proofs

A miner might consider withholding a composite proof to try releasing it strategically. However:

- With the commit–reveal protocol, the earliest valid commitment determines reward eligibility.
- Withholding a commitment delays reward eligibility rather than improving it.
- A miner cannot claim someone else’s proof because commitments are signed and timestamped.

Thus withholding provides no benefit and may reduce expected reward.

7.3.2 Sybil Attacks on Composite Rewards

Reviewers argued that miners could split their identity to receive multiple shares.
This critique misunderstands the reward structure: rewards are awarded **per composite integer m**, not per miner identity.
Submitting N identical identities yields no additional reward because only the earliest valid commit is accepted and only one reward exists per m.

Splitting identity neither increases nor decreases expected reward.

7.3.3 Search Manipulation or "Precomputation" Near the Frontier

A reviewer suggested searching for factorizations near p_n + 1,000,000, then working backward to identify potential primes. This strategy is unprofitable because:

- The next prime p_(n+1) is unpredictably close to p_n due to irregular prime gaps.
- Prime discovery requires identifying the *first* prime greater than p_n, not a later one.
- Even if a miner factors distant integers, these proofs are worthless unless they fall within the next block's interval, which is unpredictable.

Thus such manipulation offers no advantage over honest mining.

7.3.4 Withholding the Next Prime

A miner discovering p_(n+1) might consider delaying publication to hoard composite proofs. However:

- Composite proofs are already being committed by other miners.
- The miner risks losing the block reward if another miner independently finds p_(n+1).
- The chain cannot be extended without minimality; miners must supply all composite proofs.

Withholding only reduces expected reward.

7.4 Cooperative Mining Dynamics

Because composite rewards are distributed across contributors:

- miners of all sizes gain predictable returns,
- hardware centralization is mitigated,
- the mining ecosystem supports heterogeneous participants.

Large miners benefit from parallelizing composite search, but smaller miners also earn rewards for each verified factorization they contribute.

This differs from hash-based PoW, where variance is extremely high and small miners receive negligible independent returns.

7.5 Deterministic Issuance and Economic Stability

The protocol's monetary supply grows at a rate determined by the distribution of primes. The prime number theorem approximates the gap between p_n and p_(n+1) as ln(p_n), but significant variability remains. The issuance schedule is therefore:

- deterministic in sequence,

- irregular in timing,
- asymptotically sparse.

Addressing Deflation Concerns

A reviewer characterized this as a “deflationary collapse.”
This interpretation assumes that asset supply must grow rapidly to sustain usability.
However:

1. The divisibility of fractional units allows arbitrarily fine denominations.
2. Payments can use any prime fractions; value does not depend on the prime index.
3. The system’s economic model centers on **stable total value**, not inflationary expansion.

Scarcity emerges from mathematical structure, not arbitrary monetary policy.

7.6 The Role of Uniform Valuation

One of the most important clarifications introduced in this revision is that:

all fractional units, regardless of prime index, represent the same value per unit.

This principle ensures:

- fungibility across prime assets,
- straightforward transaction pricing,
- absence of market-making between primes,
- elimination of conversion problems.

Prime labels serve only to tag issuance order and do not produce economic differentiation.

7.7 Transaction Fees and Miner Incentives

Transaction fees are paid in fractional units of any prime the sender owns. This flexibility avoids issues observed in multi-asset systems in which fees must be denominated in a base asset.

Miners benefit from:

- predictable steady income from fees and composite rewards,
- independent streams of reward opportunities,
- lower variance in reward distribution.

This aligns incentives to support block production without requiring the extreme hashpower arms race observed in conventional PoW.

7.8 Game-Theoretic Interpretation

Earlier drafts used the term “Nashian” loosely, leading to criticism that no formal game-theoretic model was presented. This revision avoids such terminology unless accompanied by a complete game-theoretic treatment.

The protocol instead emphasizes **incentive dominance**:

- Honest disclosure of proofs maximizes expected reward.
- Identity splitting does not increase expected reward.

- Withholding strategies reduce expected reward.
- Front-running is prevented by commit–reveal.
- Cooperative contribution is always profitable.

These statements follow directly from protocol rules rather than requiring equilibrium analysis.

7.9 Security Against Economic Manipulation

The deterministic issuance mechanism resists manipulation because:

- No miner can select which prime to introduce.
- No miner can accelerate prime discovery with precomputation.
- No miner can gain advantage by delaying contributions.
- Prime certificates require deterministic proofs independent of miner identity.

The economic design thus preserves fairness while preventing strategic attacks on the issuance schedule.

7.10 Summary

The prime-indexed blockchain's incentive structure rewards cooperative computation while maintaining deterministic scarcity. Unlike hash-based systems, it provides predictable returns for miners of all sizes without relying on stochastic variance. The commit–reveal mechanism ensures fairness, and sparse fractional ownership supports a stable monetary system.

The revised formulation clarifies misunderstandings from earlier critiques and avoids claims unsupported by formal analysis, resulting in a more coherent and defensible economic model.

Chapter 8 — Security Considerations and Threat Analysis

Security in the prime-indexed blockchain arises from a combination of deterministic mathematics, transparent verification, post-quantum cryptographic primitives, and an incentive structure that discourages strategic misbehavior. Unlike traditional proof-of-work systems that rely on probabilistic consensus and computational asymmetry, this system grounds its security in the canonical ordering of the rational primes. The uniqueness of the next prime p_(n+1) eliminates ambiguity about which block can validly extend the chain, substantially simplifying the threat landscape.

This chapter presents a comprehensive analysis of the system's security assumptions, adversary capabilities, and protections against attacks. It addresses criticisms from external reviewers and provides corrected formulations where the original draft was ambiguous or incomplete.

8.1 Adversary Model

We consider an adversary with the following potential capabilities:

1. **Computational Resources:**
   The adversary may have significantly more computing power than honest miners.
2. **Network Access:**

The adversary may delay, reorder, or front-run messages in the peer-to-peer network.

3. **Identity Manipulation:**
   The adversary may create arbitrarily many pseudonymous identities (Sybil attacks).
4. **Cryptanalytic Capabilities:**
   The adversary may possess classical or quantum computational capabilities sufficient to compromise non–post-quantum primitives.
5. **Economic Strategy:**
   The adversary may attempt reward maximization through strategic withholding or manipulation of composite proofs.

The system must remain secure under any combination of these capabilities.

8.2 Deterministic Chain Extension and Finality

The core security property of the protocol is that **there is only one valid prime that can extend the chain at any height**. All nodes verify:

- p_(n+1) is prime,
- no prime exists between p_n and p_(n+1),
- composite proofs cover every integer in (p_n, p_(n+1)).

This has several security implications:

1. **No Alternative Branches:**
   There is no freedom to choose a different prime; therefore, miners cannot create parallel histories.
2. **No Deep Reorganizations:**
   Because there is only one correct extension, the chain tip is final as soon as a valid block is received.
3. **Low Fork Probability:**
   Temporary forks may occur if two miners broadcast blocks with the same p_(n+1) concurrently, but these forks resolve quickly and cannot persist beyond network propagation delay.

This deterministic extension rule eliminates entire classes of attacks that plague hash-based systems, such as double-spending by long-chain reorganization.

8.3 Composite Proof Security

Composite proofs demonstrate that each integer m in the required interval is not prime. The commit–reveal protocol prevents network-level interference.

8.3.1 Commit–Reveal and Front-Running Protection

An adversary cannot steal another miner’s composite proof because:

- commitments are timestamped upon network arrival,
- commitments hide the factorization using a cryptographic hash,
- only the earliest commitment corresponding to a reveal is considered valid.

Even an adversary with full visibility of the network cannot front-run or overwrite a committed proof.

8.3.2 Uniqueness of Composite Rewards
Only one composite proof per integer m is rewarded.
Splitting identity yields no benefit, eliminating the Sybil amplification vector.

8.3.3 Robustness Against DoS Attacks
Composite proofs are trivial to verify:

m == d * e

Nodes can reject malformed proofs instantly without significant resource expenditure.

8.4 Prime Certificate Security

The prime p_(n+1) must be accompanied by a deterministic certificate (e.g., ECPP, Pratt) proving primality. Nodes verify the certificate before accepting the block.

8.4.1 Resistance to Forgery
Forging a certificate requires:

- producing a false primality proof,
- or breaking the underlying mathematical assumptions of ECPP or Pratt-style certificates.

No known polynomial-time algorithms can forge such certificates. Verification is transparent and deterministic.

8.4.2 Minimality Enforcement
Skipping a prime is impossible because:

- any omitted prime q would lack a composite proof,
- blocks missing a composite proof for any m in (p_n, p_(n+1)) are invalid.

Thus certificate validity and interval completeness jointly guarantee minimality.

8.5 Cryptographic Primitives and Post-Quantum Security

Critics noted that the original draft omitted explicit signature schemes and did not address quantum safety. This revision adopts:

- **Signatures:** Dilithium (NIST PQC standard)
- **Hash function:** SHA3-256
- **Address derivation:** SHA3-256(public_key)

8.5.1 Quantum Threats
A quantum adversary running Shor's algorithm can:

- break RSA and ECDSA (but not Dilithium),
- factor integers efficiently,
- accelerate discrete-logarithm-based constructions.

However:

- **Prime search is unaffected** by Shor; it does not solve primality or next-prime discovery.
- **Grover's algorithm** provides only a quadratic speedup for search problems, which does

not threaten deterministic mining.

- **Composite proofs** remain valid: quantum factoring abilities make factoring *easier*, which does not undermine security—miners already benefit from easier factoring as part of PoW.

Thus the system is quantum-robust except for legacy algorithms, which are replaced here with PQ-safe primitives.

8.6 Network Security

The protocol uses a simplified gossip network. Security considerations include:

8.6.1 Message Flooding

Nodes can mitigate flooding attacks through:

- rate limiting,
- rejecting duplicate proofs,
- verifying commitment structure before allocating memory.

8.6.2 Message Ordering Attacks

Commit–reveal ensures that the adversary cannot profit from message delays or reordering.

8.6.3 Eclipse Attacks

Although any P2P system is vulnerable to eclipse attacks, the deterministic nature of chain extension ensures:

- an eclipsed node cannot be tricked into accepting an invalid prime or skipped prime,
- local verification will detect malformed blocks.

The attack only delays synchronization, not compromises correctness.

8.7 Economic Attack Vectors

8.7.1 Strategic Withholding of p_(n+1)

An attacker might attempt to:

- withhold discovery of p_(n+1),
- continue collecting composite proofs,
- release the prime only after accumulating advantage.

This fails because:

1. Other miners will discover p_(n+1) independently.
2. Composite proofs committed by others accumulate, reducing relative share.
3. Delaying publication risks losing the entire prime reward.

Thus withholding is strictly disadvantageous.

8.7.2 Strategic Withholding of Composite Proofs

Without commit–reveal, withholding might be profitable.
With commit–reveal:

- revealing late does not change reward share,
- withholding commits only reduces potential rewards,
- the earliest commitment wins.

Thus withholding is irrational.

8.7.3 Sybil Attacks on Composite Rewards
Because rewards are tied to integer m, not miner identity, splitting identities yields zero advantage.

8.7.4 Transaction Fee Manipulation
Fungibility of fractional units across primes prevents fee distortions:

- miners cannot demand “specific primes,”
- senders have flexibility in fee asset selection,
- no liquidity issues exist between primes.

8.8 Attacks on the Mathematical Structure

8.8.1 Attempt to Introduce a False Prime
A false prime could only be introduced by forging a deterministic certificate—a currently infeasible attack with no known polynomial-time method.

8.8.2 Attempt to Skip Primes
This is impossible because:

- the skipped integer would lack a composite proof,
- validation would fail immediately.

8.8.3 Attacks via Algebraic Extensions
Earlier drafts implied mining could extend into arbitrary rings O_K.
Reviewers correctly noted this fails when unique factorization does not hold.

This revision restricts such extensions to:

- Euclidean domains,
- Unique factorization domains (UFDs),
- Rings with explicit division algorithms.

This prevents ambiguity about irreducibles or norms and preserves security of the asset accounting model.

8.9 Limitations and Open Problems

While the system is secure under its assumptions, some areas remain open for further research:

1. **Formal Economic Equilibrium Analysis**
   A complete game-theoretic treatment remains to be written.
2. **Scalability Under Extreme Transaction Load**
   Sparse vectors scale well, but very active wallets may accumulate many entry updates.
3. **Optimized Certificate Generation**

While verification is cheap, certificate generation cost may limit prime discovery speed at scale.

None of these limitations undermine security but motivate future protocol extensions.

8.10 Summary

Security in the prime-indexed blockchain arises from deterministic issuance, efficient verification, modern cryptographic primitives, and incentive mechanisms that make strategic deviation unprofitable. Commit–reveal eliminates front-running, deterministic prime progression eliminates long-range forks, and sparse ownership representation prevents state bloat. The protocol withstands classical and quantum adversaries under reasonable assumptions, making it a robust foundation for a mathematically grounded proof-of-work system.

Chapter 9 — Post-Quantum Security and Comparative Analysis

The advent of large-scale quantum computing poses significant challenges to conventional cryptographic systems, especially those relying on integer factorization, discrete logarithms, or elliptic-curve assumptions. Many widely deployed primitives—RSA, ECDSA, and classical finite-field cryptosystems—are rendered vulnerable by Shor's algorithm, and Grover's algorithm reduces the effective strength of collision-resistant hash functions.

This chapter analyzes the prime-indexed blockchain under the post-quantum (PQ) threat model and provides a detailed comparison to classical proof-of-work systems. It incorporates corrections and clarifications raised in public reviews and expands upon the reasoning presented in the author's reply to the cryptography mailing list.

9.1 Quantum Threat Landscape

Quantum algorithms most relevant to blockchain security are:

1. **Shor's Algorithm** (polynomial-time)
   Breaks:
   - RSA
   - ECDSA
   - Diffie–Hellman
   - Discrete logarithms
   - Factorization of arbitrary integers
2. **Grover's Algorithm** (quadratic speedup)
   Affects:
   - hash preimage search
   - certain PoW difficulty functions
3. **Quantum Amplitude Amplification Techniques**
   Accelerate unstructured search, but do not fundamentally change complexity classes.

Shor's algorithm has a profound impact on most deployed blockchains: all classical

cryptocurrencies using ECDSA signatures (Bitcoin, Ethereum) become insecure as soon as scalable quantum computers exist.

However, **this protocol's core proof-of-work and consensus mechanism do not rely on hardness assumptions vulnerable to Shor or Grover**, a sharp contrast to factorization-based or discrete-log-based designs.

9.2 Post-Quantum Signatures and Addressing

The protocol adopts **Dilithium**, a lattice-based, NIST-standard post-quantum signature scheme. This addresses the primary quantum vulnerability of classical blockchains: signature forgery.

Why Dilithium?

- Resistant to known quantum attacks
- Mature, rigorously analyzed lattice construction
- Efficient verification
- Supported by standardized tooling and future hardware acceleration

Addresses are derived as:

address = SHA3_256(public_key)

This provides quantum-robust preimage resistance (Grover only halves the effective bit security; SHA3-256 retains ≈128-bit strength).

Thus quantum threat to wallet confidentiality or control is effectively mitigated.

9.3 Quantum Impact on Prime Search

A key point clarified in my reply email — and misunderstood by reviewers — is that *quantum computing does not accelerate prime search*.

9.3.1 Shor's Algorithm Does Not Find Primes

Shor factors arbitrary integers N, but the mining process in this protocol does not involve:

- factoring N−1 or N+1 for primality testing,
- searching over structured integers for special forms,
- using cryptographic assumptions about difficulty.

Mining consists of checking whether each integer m is prime or composite, in sequence.

Shor gives **no advantage** for:

- primality testing
- next-prime search
- certificate generation (which relies on structures not sped up by Shor)

9.3.2 Grover's Algorithm Gives Only Quadratic Speedup

Grover can accelerate unstructured search for a prime by at most a square root factor. However:

- The protocol requires checking integers in *fixed sequence*, not searching a large unordered domain.
- A quadratic speedup simply accelerates all miners proportionally; difficulty adjusts naturally

via prime gaps.

- Even with quadratic speedup, the chain remains far below prime sizes that strain certificate generation.

Conclusion:
Quantum computing does not undermine the mathematical basis or fairness of the mining process.

9.4 Quantum Impact on Composite Proofs

Composite proofs consist of finding d, e such that:

m = d * e.

A quantum adversary can factor m more quickly using Shor's algorithm. While this accelerates composite proof production, it does **not reduce security**, because:

1. **Composite proofs are not security-critical.**
   They only certify that m is composite; they do not rely on hard-to-find factors for correctness.
2. **Easier composite proofs simply shift difficulty back toward prime discovery.**
   Miners with quantum capability produce composite proofs faster, but:
   - rewards adjust proportionally,
   - honest miners without quantum hardware can still find smaller factors via classical methods,
   - economic fairness follows from deterministic reward rules.
3. **Quantum factoring offers no attack vector.**
   It cannot forge certificates, skip primes, or invalidate correctness rules.

Thus quantum factoring changes **mining economics**, not security.

9.5 Quantum Impact on Primality Certificates

The protocol uses deterministic prime certificates such as ECPP or Pratt-style proofs. Verification requires:

- checking elliptic-curve conditions (ECPP)
- verifying recursively the primality of smaller integers (Pratt)

Quantum capabilities do not break these certificate systems. Shor's algorithm helps *factoring*, but primality proofs do not require concealing factors — they require *revealing* algebraic structure.

9.5.1 Verification Remains Cheap

Prime certificate verification is polynomial-time in log(p).
Even with cryptographically large primes:

- verification is fast,
- certificate creation remains feasible for all primes the chain will ever reach.

Given realistic block rates, the chain never approaches primes whose certificate generation would exceed practical bounds.

9.6 Attack Surfaces Under Quantum Models

9.6.1 Signature Forgery
Mitigation: post-quantum Dilithium signatures.
Result: attacker cannot steal wallets or inject fraudulent transactions.

9.6.2 Composite Proof Manipulation
Quantum speedup gives no advantage in:

- front-running (commit–reveal prevents this),
- stealing contributions (signatures bind identity),
- overwriting proofs (only earliest valid commit counts).

9.6.3 Prime Forgery / Skipping Primes
Cannot occur without forging a deterministic certificate — presently infeasible even for quantum adversaries.

9.6.4 Replay Attacks and Network Interception
Mitigated by:

- canonical block hashing,
- commit–reveal timestamps,
- deterministic ordering rules.

Quantum adversaries do not gain network-level advantage beyond increased bandwidth.

9.6.5 State Manipulation
Nodes independently recompute:

- wallet balances,
- composite proof sets,
- sparse vectors,
- Merkle roots.

Tampering requires breaking SHA3-256 or Dilithium — both PQ-resilient.

9.7 Comparison With Classical Blockchains Under PQ Threat

Below is a high-level comparison:

Bitcoin / Ethereum (ECDSA + hash-based PoW)
- **Signature layer:** completely broken by Shor.
- **Mining layer:** hash search is quadratic-weakened under Grover (effective 2× difficulty reduction).
- **Consensus:** probabilistic; vulnerable to reorg attacks if adversary has quantum advantage.

Prime-Indexed Blockchain
- **Signature layer:** PQ-safe (Dilithium).

- **Mining layer:** unaffected by Shor; Grover provides marginal speedup.
- **Consensus:** deterministic; does not allow reorganization.
- **Composite proofs:** easier under quantum computing but security-neutral.
- **Prime certificates:** unaffected by quantum algorithms.

Thus the protocol's *foundational structure* complements PQ assumptions better than classical PoW systems.

9.8 Relation to PQ-Robust Proof-of-Work Literature

Verifiable Delay Functions (VDFs)

VDFs require sequential computation but often rely on algebraic groups vulnerable to quantum attacks (unless carefully designed). The prime-indexed protocol achieves:

- determinism,
- sequential nature,
- verifiable certificates

without relying on trapdoor groups.

Useful Proof-of-Work

Prime discovery and composite certification represent a form of useful computation aligned with mathematical classification tasks. Unlike many "useful PoW" proposals:

- work is deterministic,
- verifiable in polynomial time,
- independent contributors can collaborate without trust.

Proof-of-Space / Proof-of-Storage

Often rely on hash-based constructions weakened by Grover.
The prime-indexed system does not rely on large hash-space exploration.

Overall, the protocol fits naturally into the landscape of PQ-resilient, deterministic consensus mechanisms.

9.9 Long-Term PQ Viability

In extremely advanced quantum scenarios — beyond current scientific projections — adversaries may gain:

- near-instant factoring of arbitrary integers,
- quantum acceleration of primality testing via advanced algorithms not yet known,
- polynomial (or super-polynomial) shortcuts for certificate generation.

Even under these assumptions, the protocol's core remains surprisingly robust:

- deterministic prime progression cannot be altered,
- verification remains trivial for nodes,
- consensus does not rely on hardness assumptions,

- quantum advantage affects *efficiency*, not *security*.

Mining economics may change, but consensus correctness does not.

9.10 Summary

The prime-indexed blockchain exhibits strong post-quantum security properties:

- **PQ-safe signatures** eliminate wallet forgery risks.
- **Prime discovery** is intrinsically resistant to quantum acceleration.
- **Composite proofs** benefit from quantum speedups without reducing security.
- **Deterministic consensus** prevents reorganization attacks even under extreme adversarial advantage.
- **Hashing with SHA3** remains secure against known quantum threats.
- **Prime certificate systems** retain validity independent of quantum factoring breakthroughs.

In contrast to classical blockchains whose foundations (ECDSA, RSA, hash difficulty scaling) are severely weakened by quantum computing, this system's core computational and mathematical structure is inherently resilient.

Thus the protocol provides a credible pathway toward **future-proof consensus** in a world where quantum computation becomes practical.

Chapter 10 — Comparison With Primecoin, Factor-Based Blockchains, and Related Proof-of-Work Designs

This chapter situates the prime-indexed blockchain within the broader family of number-theoretic proof-of-work systems. In particular, it compares the protocol to:

1. **Primecoin (King, 2013)** — the first cryptocurrency to use prime-related structures for PoW;
2. **Factor-chain mining proposals** — systems that use integer factorizations as PoW;
3. **Other "useful proof-of-work" and verifiable computation proposals** — systems seeking productive computation outside classical hashing.

The comparisons clarify philosophical differences, security assumptions, verification costs, mining incentives, and scalability. Reviewer feedback highlighted the absence of such a comparison in the original draft; this revised chapter addresses that gap comprehensively.

10.1 Primecoin: Chain of Primes vs. Chain of Prime Indices

Primecoin was the first deployed cryptocurrency based on prime numbers. Its proof-of-work searches for specific structured prime constellations such as Cunningham chains and bi-twin chains.

10.1.1 Primecoin Proof-of-Work

Primecoin miners attempt to discover prime sequences of the form:

- p, 2p + 1, 4p + 1, ... (Cunningham chains), or
- related structured sequences.

The number of consecutive primes in the chain determines the block difficulty. Block rewards are paid to miners who discover longer or sufficiently rare chains.

10.1.2 Limitations of Primecoin (as identified over a decade of analysis)

1. **Probabilistic difficulty:**
   Difficulty depends on the rarity of specific chain types. This results in:
   - unstable block times,
   - difficulty adjustment challenges,
   - irregular issuance.
2. **Non-deterministic chain extension:**
   Hash-based competition determines the winner; multiple valid prime chains may exist simultaneously.
3. **Verification cost increases:**
   Verifying long prime chains is inexpensive but still grows with chain length.
4. **No cooperative PoW:**
   All rewards go to a single miner; intermediate partial results are discarded.
5. **Limited economic interpretation:**
   Primecoin does not create prime-indexed assets or fractional holdings.
   It simply uses primes as a mining puzzle.

10.1.3 How the Prime-Indexed Blockchain Differs Fundamentally

Feature	Primecoin	Prime-Indexed Blockchain
Consensus structure	Probabilistic, hash-dominant	Deterministic next prime p_(n+1)
Proof-of-work objective	Find rare structured chains	Certify every integer between consecutive primes
Verification model	Chain-length dependent	Uniform, O(1) per composite
Rewards	Winner-takes-all	Shared between prime finder + composite provers
Asset system	None	Fractional ownership of each prime
Determinism	Low	High (unique block per height)
Finality	Probabilistic	Immediate
Cooperative mining	No	Yes

In essence, **Primecoin uses primes as randomness; this protocol uses primes as a deterministic backbone for consensus, issuance, and asset structure.**

10.2 Comparison to Factor-Chain Blockchains

Several proposals (none widely deployed) attempted to use **integer factorization** as proof-of-work. The typical design involves:

- selecting a random large integer N,
- requiring miners to factor N or provide partial factorizations,
- verifying correctness using multiplication.

Reviewers mistakenly associated suggested protocol with this class; this chapter clarifies the separation.

10.2.1 Common Factor-Based PoW Designs

Such systems usually involve:

- selecting integers via hash function,
- requiring partial factorization or smoothness proofs,
- awarding mining rights based on factorization difficulty.

10.2.2 Fundamental Weaknesses of Factor-Based PoW

1. **Quantum fragility:**
   Shor's algorithm trivializes factorization.
2. **Trapdoor risk:**
   A malicious party generating N could embed hidden structure (safe primes, smooth composites).
3. **Need for random N:**
   Choosing N deterministically breaks PoW unpredictability; choosing N randomly increases block variance.
4. **Difficulty unpredictability:**
   Factoring random numbers varies dramatically in hardness.
5. **Verification complexity:**
   Some schemes require repeated validation or heuristic assumptions about smoothness.

10.2.3 Why the Prime-Indexed Blockchain is Not a Factor-Chain System

Suggested protocol:

- never generates arbitrary composite N;
- never requires factoring arbitrary N;
- never relies on factorization hardness;
- never uses factoring as a security assumption.

Instead:

- factoring is "bonus work" for composite proofs;
- factoring difficulty does not underpin security;
- verification requires only multiplication (m = d * e), not reverse factorization;
- quantum factoring does not break the protocol.

Thus the protocol belongs to a different conceptual family: **deterministic sequential certification of the integer line**.

10.2.4 Strengths Compared to Factor-Chain PoW

Aspect	Factor-Chain Proposals	Prime-Indexed Blockchain
Security Foundation	Factorization hardness	Deterministic arithmetic + primality
Quantum risk	Catastrophic	Minor (only affects composite efficiency)
Trapdoor possibility	High	None
Consensus model	Probabilistic	Deterministic
Verification	Variable cost	Constant-time multiplication
Economic model	Winner-takes-all, no structure	Fractional prime-indexed assets

This shows that the prime-indexed blockchain is unique in combining:

- deterministic consensus,
- number-theoretic grounding,
- cooperative contribution,
- PQ resilience,
- simple verification,
- mathematically transparent issuance.

No existing system — whether hash-based, prime-based, factor-based, or VDF-based — offers the same combination of properties.

10.6 Philosophical Distinction: Primes as Ledger vs. Primes as Puzzle

The most important conceptual difference is:

- **Primecoin** uses primes *as puzzles* to be found probabilistically.
- **Factor-blockchains** use primes *as hardness assumptions* (factorization).
- **Suggested protocol** uses primes *as a deterministic index for issuance, consensus, and asset structure*.

The prime sequence itself becomes the backbone of the ledger:
each block corresponds to a universally defined mathematical object (the next prime), making the blockchain inherently deterministic and unverifiable shortcuts impossible.

This shift in perspective distinguishes the system from all prior number-theoretic PoW designs.

10.7 Summary

The prime-indexed blockchain stands apart from previous prime-related or integer-based proof-of-work systems. Compared to Primecoin, it replaces probabilistic chain competition with deterministic progression and introduces a fractional prime asset economy. Compared to factor-based proposals, it eliminates reliance on hardness assumptions and trapdoor risks. Compared to useful-PoW and VDF proposals, it preserves simple verification while maintaining unambiguous correctness.

The result is a mathematically structured, post-quantum-resilient, cooperative-mining protocol that does not resemble any previously deployed or proposed blockchain architecture.

Chapter 11 — Game-Theoretic Analysis and Nash Equilibrium Properties

This chapter provides a first attempt to formalize the cooperative mining mechanism of the prime-indexed blockchain in terms of non-cooperative game theory, in the spirit of Nash's framework. Earlier drafts used the term "Nashian" informally, which rightly attracted criticism. Here we explicitly define players, strategies, utility functions, and equilibria, and we show under which assumptions the **honest cooperative strategy profile** constitutes a Nash equilibrium, or is at least "close" to one in a well-defined sense.

The goal is not to claim a fully exhaustive equilibrium analysis under all possible deviations, but to demonstrate that the core incentives of the protocol are compatible with Nash's concept of equilibrium in a stylized model.

11.1 Players, Actions, and Stages

We model the system as a repeated game with the following elements.

11.1.1 Players

- Each **miner** is a player i in a set of miners M = {1, 2, ..., N}.
- Miners have heterogeneous computational resources (CPU, GPU, ASIC, quantum hardware, etc.).
- Nodes that do not mine are not modeled as strategic players; they act as deterministic validators.

11.1.2 Stages

The game repeats over **block intervals**. In each block interval n:

- The frontier prime is p_n.
- Players search for composite proofs for integers m in (p_n, p_(n+1)) and for the next prime p_(n+1) itself.
- At the end of the interval, exactly one valid block can be produced.

We treat each block interval as a **stage game**, and the long-run protocol as the infinite repetition of this stage game.

11.1.3 Strategy Space (Simplified)

For each stage game, we abstract away details and define three core choices for each miner i:

1. **Honest strategy H**
    - Immediately commit and later reveal any composite proof the miner finds.
    - Immediately broadcast the prime certificate for p_(n+1) if discovered.
    - Use the protocol's commit–reveal scheme correctly.
    - Do not submit duplicate or invalid proofs.

2. **Deviating strategy D1: Withhold composite proofs**
    - Delay or selectively reveal composite proofs in an attempt to improve reward.
3. **Deviating strategy D2: Identity splitting / Sybil splitting**
    - Split the miner's identity into multiple addresses and try to claim multiple reward shares.
4. **Deviating strategy D3: Front-running and plagiarism**
    - Attempt to re-broadcast other miners' proofs as one's own.
5. **Deviating strategy D4: Withhold the next prime**
    - Discover p_(n+1) but delay publishing it to collect alleged advantages.

In reality, the strategy space is much richer, but if we can show that **no unilateral deviation among these key classes improves expected payoff**, then the honest profile H is at least a Nash equilibrium in this restricted game.

11.2 Utility Functions

We define each miner's utility in a given stage as the **expected value of rewards** they receive minus the **cost of computation**.

Let:

- R_prime(n) be the fraction of the newly minted prime p_(n+1) allocated to the prime finder in block n.
- R_composite(n) be the total fraction allocated to composite proof providers in block n (including transaction fees).
- C_i(n) be the cost incurred by miner i in computational effort (energy, time, hardware depreciation) during the block interval n.
- S_i(n) be the set of integers m for which miner i provides the **first valid commitment** and later valid reveal.
- P_i(n) be the event that miner i is the first to discover and publish a valid prime certificate for p_(n+1).

Then the **expected utility** of miner i in stage n under a given joint strategy profile s is:

U_i(n | s) = E[
R_prime(n) * I(P_i(n))

- sum over m in S_i(n) of (R_composite(n) / K_n)
- C_i(n)
  ],

where:

- I(P_i(n)) is 1 if i discovers and publishes the prime first; otherwise 0.
- K_n is the total number of composite integers in (p_n, p_(n+1)) (or a variant that weights by difficulty).
- Expectations are taken over randomness in computation, network delays, and prime gap size.

Over the infinitely repeated game, the miner's total utility is:

U_i_total = sum over n of δ^(n-1) * U_i(n | s),

where 0 < δ < 1 is a discount factor representing time preference.

11.3 Honest Strategy Profile

In the **honest strategy profile H = (H_1, ..., H_N)**:

- Every miner publishes commitments for composite proofs as soon as they are found.
- Every miner reveals proofs according to protocol rules.
- Any discovered prime p_(n+1) is broadcast promptly with its certificate.
- No miner attempts identity splitting for additional reward.
- No miner attempts to front-run others' proofs.

Under H:

- Each miner's expected reward is proportional to their share of useful computation.
- No reward is wasted, and every valid composite proof is eligible for a share of R_composite(n).
- Composite proofs are not duplicated, and the earliest commitment per m wins.

In such a setting, we want to show: **no miner can increase U_i_total by unilaterally deviating from H, given that all others follow H.** That is the definition of a Nash equilibrium.

11.4 Analysis of Deviating Strategies

We now analyze each deviation type separately and contrast its expected payoff with that of honest behavior.

11.4.1 Deviation D1: Withholding Composite Proofs

In D1, miner i withholds some composite proofs instead of committing immediately. Intuitively, if rewarded per-composite, why not reveal later?

Under the protocol:

- Reward for each m is **bounded and fixed**: at most R_composite(n) / K_n.
- The earliest valid commitment wins.
- Revealing earlier does not reduce reward; it only guarantees eligibility.

Let Δ_i be the change in expected composite reward if miner i withholds composite proofs instead of committing honestly. Because:

- withholding does not increase the reward share for a given m,
- withholding increases the risk that another miner will discover and commit the proof first,
- the protocol does not reward "being last" or "revealing at block time" — only being first,

we obtain:

E[Δ_i | D1] <= 0.

At best, withholding yields the same expected reward (in the unrealistic case where no other miner ever finds m), and in all realistic competitive environments it strictly lowers expected reward.

Hence D1 is **not a profitable deviation**. In utility terms:

U_i(n | D1, H_-i) <= U_i(n | H_i, H_-i),

with strict inequality whenever there is nonzero competition on composite proofs.

11.4.2 Deviation D2: Identity Splitting (Sybil)

Suppose miner i splits into k identities i_1, ..., i_k and attempts to claim more composite rewards. However, rewards are **per integer m**, not per identity. The protocol grants exactly one reward share for each composite, to the earliest valid commitment.

Let:

- S_i be the set of m for which i (or its identities) is the earliest committer.
- R(m) be the reward for m (e.g., R_composite(n) / K_n).

Under honest identity (no splitting):

U_i(n | single_id) = sum over m in S_i of R(m) - C_i(n).

Under identity splitting:

- The same physical resources produce the same set S_i of proofs (the computer's work remains identical).
- Only the earliest identity among (i_1, ..., i_k) per m gets the reward; others are ignored.
- Total reward per m is still exactly R(m).

Thus:

U_i(n | split) = U_i(n | single_id) - cost_of_managing_extra_ids.

Therefore:

U_i(n | D2, H_-i) <= U_i(n | H_i, H_-i),

and in practice, strict inequality holds because extra identities can only add overhead, not additional reward.

Identity splitting is **not a profitable deviation**.

11.4.3 Deviation D3: Front-Running and Plagiarism

In D3, miner i attempts to steal another miner's composite proof by broadcasting it as their own. Without commit–reveal this might be profitable; with commit–reveal:

- Each composite proof is tied to a prior commitment: hash(m || d || address || nonce).
- The earliest valid commitment is credited the reward.
- A plagiarist cannot retroactively generate a matching commitment after seeing the proof.

Hence:

- If i did not commit first, plagiarized proof yields no reward.
- If i did commit first, this is equivalent to honest behavior.

Thus front-running is either impossible or reduces to honest mining.

So:

U_i(n | D3, H_-i) <= U_i(n | H_i, H_-i),

with equality only in trivial cases.

11.4.4 Deviation D4: Withholding the Next Prime p_(n+1)

In D4, miner i discovers p_(n+1) but delays publishing it, hoping to collect more advantages (e.g., more composite proofs, or a better position in the network).

Under the protocol:

- Composite rewards are independent of who publishes the prime.
- Other miners may also discover p_(n+1); first valid broadcast wins R_prime(n).
- No “extra” reward is given for withholding.

Let:

- P_honest be the probability that i is first to publish p_(n+1) under honest behavior.
- P_withhold be the corresponding probability under withholding.

Withholding cannot **increase** P_withhold relative to P_honest; if anything, it reduces it because:

- During the delay, other miners continue searching.
- Once they find p_(n+1) and broadcast, i’s chance to win R_prime(n) becomes zero.

Thus the expected change in prime reward is:

E[Δ_i_prime | D4] <= 0.

Composite rewards do not improve either: others are still committing proofs during the delay.

Hence:

U_i(n | D4, H_-i) <= U_i(n | H_i, H_-i),

with strict inequality if other miners are active.

11.5 Honest Mining as a Nash Equilibrium (Restricted Model)

Within this simplified model:

- Strategy space S_i = {H, D1, D2, D3, D4};
- Utility U_i(n | s) defined as above.

We have shown:

For every miner i and for each deviation type Dk in {D1, D2, D3, D4}:

U_i(n | Dk, H_-i) <= U_i(n | H_i, H_-i).

In other words, given that all other miners follow the honest strategy, miner i does **not** increase expected utility by deviating to any of these deviations. Therefore, under this restricted strategy set:

The honest strategy profile H is a Nash equilibrium of the stage game.

In the repeated game (over infinitely many blocks) with discount factor δ, if miners are myopic or have stationary preferences, this equilibrium extends naturally to the repeated setting: at each stage, no unilateral deviation improves discounted utility.

11.6 Limitations of the Analysis

The analysis above is intentionally conservative and does **not** claim that:

- we have enumerated all possible strategies,

- the equilibrium is unique,
- the equilibrium is robust under coalitions or complex information structures.

Some open questions include:

1. **Coalition Deviations:**
   Multiple miners might coordinate to share information or selectively withhold proofs. The protocol's fairness properties suggest limited gain, but a full coalition-resilient equilibrium analysis is beyond this chapter's scope.
2. **Network Topology and Latency:**
   The model assumes symmetric propagation delay; in reality, miners with better connectivity may enjoy small advantages. This does not break equilibrium but could affect distribution of rewards.
3. **Advanced Quantum Deviations:**
   A miner with significantly superior hardware may achieve greater share of composite proofs or faster prime discovery; economic equilibrium must adapt to such asymmetries.
4. **Risk Preferences:**
   We modeled miners as risk-neutral. Risk-averse or risk-seeking behaviors could modify preferences over low-variance cooperative rewards vs. high-variance solo strategies.

Thus, we describe the demonstrated result as:

> **The honest cooperative strategy profile is a Nash equilibrium in a natural restricted game model of the protocol, covering the main deviation types raised in critiques.**

Further work can extend this analysis with richer strategy spaces and more detailed modeling of network and coalition structures.

11.7 Conceptual Interpretation: "Nashian" Cooperation

Within the limits above, the protocol exhibits the following "Nashian" properties:

1. **Incentive Compatibility:**
   Honest behavior maximizes expected utility given honest behavior by others.
2. **Reward Alignment:**
   Every unit of useful work has a proportional, verifiable reward.
3. **No Profitable Deviations in Key Dimensions:**
   Withholding, front-running, identity splitting, and prime withholding are all strictly or weakly dominated by honesty.
4. **Cooperative Equilibrium:**
   Miners coexist in a cooperative equilibrium where independent contributions are rewarded without requiring trust or central coordination.

In this precise sense, the protocol justifies describing its incentive structure as **Nashian**: behavior that contributes honestly to the common mathematical objective is a best response given others' honesty.

11.8 Summary

This chapter formalizes the mining and reward mechanism of the prime-indexed blockchain as a non-cooperative game. Under reasonable modeling assumptions and a restricted but meaningful

strategy set, honest cooperative mining emerges as a Nash equilibrium: no miner can benefit by unilaterally deviating to withholding, identity splitting, front-running, or prime withholding.

While the analysis is not exhaustive, it provides a rigorous first step toward grounding the protocol's design in Nash's equilibrium framework, correcting earlier informal usage and clarifying the sense in which the system's proof-of-work is "Nashian."

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